WS#10-4
Matrix Algebra

1. **Matrix Usage**

In a survey of 900 people, the following information was obtained:

200 males	Thought federal defense spending was too high
150 males	Thought federal defense spending was too low
45 males	Had no opinion
315 females	Thought federal defense spending was too high
125 females	Thought federal defense spending was too low
65 females	Had no opinion

This data can be arranged in a rectangular array as follows:

	Too High	Too Low	No Opinion
Male	200	150	45
Female	315	125	65

Or as a matrix as:

$$\begin{bmatrix} 200 & 150 & 45 \\ 315 & 125 & 65 \end{bmatrix}$$

2. **Definitions and properties**

You will be responsible to read the section completely and review the definitions and properties of the following:

A. Dimensions

B. Square Matrix

C. Equal Matrices

D. Commutative Property

1. Addition
2. Multiplication

E. Associative Property

1. Addition
2. Multiplication

F. Zero Matrix

G. Scaler Multiplication

List properties

1.
2.
3.

H. Column Vector

I. Row Vector

J. Distributive Property

K. Identity Property

L. Inverse Property

- When adding or subtracting matrices, their dimensions must be the same (or no solution)
- When multiplying matrices, the # of columns in A must equal the # of rows in B

$$\begin{matrix} A & & B \\ m \times r & & r \times n \end{matrix}$$

→ inside values must be the same in order to multiply. results in an $\underline{m \times n}$ matrix.

$I_2 = 2 \times 2$ Identity Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I_3 = 3 \times 3$ Identity Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. If $A = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 0 \\ -2 & 3 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 4 & 1 \\ 6 & 2 \\ -2 & 3 \end{bmatrix}$, perform the operations below:

A. A – B

$$\begin{bmatrix} 0-4 & 3-1 & -5-0 \\ 1-(-2) & 2-3 & 6-(-2) \end{bmatrix} = \begin{bmatrix} -4 & 2 & -5 \\ 3 & -1 & 8 \end{bmatrix}$$

B. 2A + 4B

$$\underbrace{\begin{bmatrix} 0 & 6 & -10 \\ 2 & 4 & 12 \end{bmatrix}}_{2A} + \underbrace{\begin{bmatrix} 16 & 4 & 0 \\ -8 & 12 & -8 \end{bmatrix}}_{4B} = \begin{bmatrix} 16 & 10 & -10 \\ -6 & 16 & 4 \end{bmatrix}$$

C. CB

$$\begin{matrix} C & & B \\ 3 \times 2 & & 2 \times 3 \end{matrix} \rightarrow \begin{matrix} CB \\ 3 \times 3 \end{matrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 4(4)+1(-2) & 4(1)+1(3) & 4(0)+1(-2) \\ 6(4)+2(-2) & 6(1)+2(3) & 6(0)+2(-2) \\ -2(4)+3(-2) & -2(1)+3(3) & -2(0)+3(-2) \end{bmatrix}$$

$$\begin{bmatrix} 14 & 7 & -2 \\ 20 & 12 & -4 \\ -14 & 7 & -6 \end{bmatrix}$$

D. CA + 5I₃

$$\underbrace{\begin{bmatrix} 4(0)+1(1) & 4(3)+1(2) & 4(-5)+1(6) \\ 6(0)+2(1) & 6(3)+2(2) & 6(-5)+2(6) \\ -2(0)+3(1) & -2(3)+3(2) & -2(-5)+3(6) \end{bmatrix}}_{CA} + 5\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I_3} = \begin{bmatrix} 6 & 14 & -14 \\ 2 & 27 & -18 \\ 3 & 0 & 33 \end{bmatrix}$$

4. Manually, Find the inverse of

$A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

- If Det of the matrix = 0, then that matrix has no inverse.

A. ~~Manual procedure:~~ Def. Let A be a square n×n matrix. If there exists an n×n matrix A^{-1}, read "A inverse", for which $A \cdot A^{-1} = A^{-1} \cdot A = I_n$ then A^{-1} is called the inverse of matrix A.

✓ A^{-1} =

$$\frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

✓ B^{-1} =

No inverse b/c Det(B) = 0

- Rule to find inv. of a 2×2 matrix

if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{\text{Det}(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

5. Use the inverse matrix to solve: $\begin{cases} x + y = 3 \\ -x + 3y + 4z = -3 \\ 4y + 3z = 2 \end{cases}$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

$$A^{-1} \cdot B = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \begin{matrix} \rightarrow x \\ \rightarrow y \\ \rightarrow z \end{matrix}$$

$x = 1$
$y = 2$
$z = -2$

* To solve systems using inverses, let A = matrix made up of the lead coefficient of the variables on the (L) side of the equal sign and B = constants on the (R) side of the equal sign

$\boxed{\text{In your Calc, Do } A^{-1} \cdot B}$